

# of Space-Time ?

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## Abstract

The Finsler-relativistic metric function  $F(g; R)$  and the associated Hamiltonian function  $H(g; P)$ , being considered together with explicit Finslerian special-relativistic kinematic transformations, give rise to a self-consistent and rigorous framework upon which corrections to Lorentz-relativistic quantities can properly be evaluated. The concomitant relations generalize their Lorentzian prototypes through the presence of a single characteristic parameter,  $g$ , so that the explicated Particle-Antiparticle Asymmetry, as well as the search for possible distinction between the pseudo-Euclidean Light Geometry and the Finsler-relativistic Neutrino Geometry, can well be traced in terms of this parameter. At any fixed rest-mass value  $m$  and  $g \neq 0$ , the dependences of energy on three-dimensional momentum prove to be of different forms, as given by the respective functions  $E^{(+)}(g; m; |\mathbf{P}|)$  and  $E^{(-)}(g; m; |\mathbf{P}|)$ , for particles and antiparticles. This splitting of the mass-shell, as well as various entailed Finslerian approximations with respect to  $g$ , can naturally be proposed for experimental study in order to obtain estimations on  $g$ . Since neutrinos and antineutrinos are uncomposed neat particles, measuring the difference between their velocities seems to be the best way for testing implied Finslerian corrections to conventional Lorentzian quantities. Accelerators which can produce neutrinos and antineutrinos are ideal instruments to gain this aim. The known measurements led to the conclusion that the difference  $< 0.7 \cdot 10^{-4}$  (95% *CL*), while announced future long baseline neutrino experiments probably raise the sensitivity to approach the high level of  $\sim 10^{-9}$ .

## 1. Introduction

**1.1. Historical notes** in front of any relativistic approach should trace back to the classical Einstein work of 1905 [1] (see also [2–3]), in which the Special Theory of Relativity (STR) was founded upon use of the postulate of light velocity invariance, and to the great event of 1908 when Minkowski reported the adequate relativistic four-dimensional space-time geometry based on the pseudo-Euclidean metric function  $\sqrt{T^2 - X^2}$  (see [4–5]). The next step of fundamental theoretical significance for STR was made in the work by Frank and Rothe [6], where the authors argued that the assumption of the existence of invariant velocity is not necessary in order to arrive at the correct transformations. Since then, various analytical and conceptual aspects of the Lorentz-invariant STR have been analyzed in numerous keen works (see [7–60] and references therein; many updated references can be found in the recent conference volume [61]).

In the domain of particles, the early notes on possible violation of the STR were made by L.B. Redei, when considering the muon lifetime [62–64]. The “Redei behaviour” of spectra was taken into account in an extended work by H.B.Nielsen and coauthors [65–67] devoted to non-Lorentzian aspects. In this connection, the work by J.Ellis, M.K.Gailard, D.V.Nanopoulos and S.Rudaz [68] should also be considered. The recent work by S.Coleman and S.L.Glashow [69] (continued in [70]) concerned with possible neutrino tests of the STR would attract new attention of researchers to possible deeper origin of relativistic invariance.

However, the investigators did not invoke the Finsler geometry tools to extend the pseudo-Euclidean relativistic framework, so that the metric function  $\sqrt{T^2 - X^2}$  has appeared to be “*a relativistic invariant of nine decades elapsed since 1908 year*”.

**1.2. Nowadays reasoning** should note, first of all, that each researcher who makes an attempt to lift the habitual laws and concepts of the present-day relativistic physics to the advanced-Finslerian level faces the necessity to explain in what way he intends to generalize the ordinary Lorentz-invariance. On the other hand, the Lorentz-transformations are actually predetermined by the pseudo-Riemannian patterns adopted to metricize the space-time base manifold. Therefore, the very possibility of the Euclidean-to-Finslerian extension of the Lorentz-invariant relativistic theories depends crucially on whether the Finsler-relativistic geometry for the space-time can be constructed in a consistent and trustworthy way. It proves that the latter-type geometry can well be founded upon the use of the Finslerian metric function (FMF)  $F(g; R)$  presented below and of the associated Finslerian metric tensor (FMT)  $g_{pq}(g; R)$  [71–88].

**1.3. The uniqueness theorem for the special-relativistic FMF** comes from a due attentive consideration. Indeed, let  $M$  be the background four-dimensional space-time,  $R \equiv \{R^p\} \in M; p, q, \dots = 0, 1, 2, 3$ . Any FMF  $F(R)$  defines the hypersurface  $\mathcal{I} = \{R \in M : F(R) = 1\}$  called conventionally *the indicatrix* (see [89–94]). The well-known fact is that the pseudo-Euclidean geometry may be characterized by the condition that the associated indicatrix is a pseudosphere of radius 1 (a hyperboloid), and hence is a space of constant curvature  $-1$ . The case founded on the property

(P1) *The indicatrix is a space of constant negative curvature  $R_{\mathcal{I}} \neq -1$*

can be regarded as the nearest Euclidean-to-Finslerian relativistic generalization. Also, we assume the properties

(P2) *The FMF is compatible with the principle of spatial isotropy (the  $\mathcal{P}$ -parity),*

(P3) *The associated FMT is of the time-space signature  $(+ - - -)$ ,*

and

(P4) *The principle of correspondence holds true,*

that is, the associated FMT reduces exactly to its ordinary known special-relativistic prototypes when  $R_{\mathcal{I}} \rightarrow -1$ , which physical significance is quite transparent.

All the items (P1)–(P4) are obeyed whenever the choice  $F = F(g; R)$  is made, in which case

$$R_{\mathcal{I}} = - \left( 1 + \frac{1}{4}g^2 \right).$$

Vice versa, we can claim the following

**THEOREM** *The properties (P1)–(P4), when treated as conditions imposed on the FMF, specify it unambiguously in the form  $F = F(g; R)$ . For more deatil, the reader is referred to [87].*

#### 1.4. The special-relativistic FMF

$$F(g; R) = |T + g_- |\mathbf{R}||^{G_+/2} |T + g_+ |\mathbf{R}||^{-G_-/2}$$

can be adduced by the Hamiltonian function

$$H(g; P) = \left| P_0 - \frac{|\mathbf{P}|}{g^+} \right|^{G_+/2} \left| P_0 - \frac{|\mathbf{P}|}{g^-} \right|^{-G_-/2},$$

where the following notation has been used:

$$h \stackrel{\text{def}}{=} \sqrt{1 + \frac{1}{4}g^2},$$

$$g_+ = -\frac{1}{2}g + h, \quad g_- = -\frac{1}{2}g - h,$$

$$g^+ = 1/g_+ = -g_-, \quad g^- = 1/g_- = -g_+,$$

$$g^+ = \frac{1}{2}g + h, \quad g^- = \frac{1}{2}g - h.$$

$$G_+ = g_+/h, \quad G_- = g_-/h,$$

$$G^+ = g^+/h, \quad G^- = g^-/h,$$

together with  $R^0 = T$ . Notice that

$$g_+ \stackrel{g^+ \rightarrow -g}{\Longleftrightarrow} -g_-, \quad g^+ \stackrel{g^+ \rightarrow -g}{\Longleftrightarrow} -g^-, \quad G_+ \stackrel{g^+ \rightarrow -g}{\Longleftrightarrow} -G_-, \quad G^+ \stackrel{g^+ \rightarrow -g}{\Longleftrightarrow} -G^-.$$

Various relativistic consequence of use of these  $F(g; R)$  and  $H(g; P)$  have been studied in [71–88].

**1.5. The characteristic Finslerian parameter  $g$**  comes directly from the indicatrix curvature  $R_{\mathcal{I}}$ . The question as to what is the physical sense and meaning of the parameter  $g$  can be answered by focusing attention on varios lucid Finsier-relativistic

relations, thereby giving rise to quite a number of fairly unexpected juxtapositions. For instance, noting the formulae given below in Section 2, we can conclude that there are no Finslerian corrections to the velocity-momentum transition if and only if there are no Finslerian corrections to the law of addition of relativistic velocities.

Characteristically, the difference of the indicatrix curvature from  $-1$  shows itself beginning with the second order of the parameter  $g$ , whereas the FMT components  $g_{pq}(g; R)$  start differing from the pseudo-Euclidean  $\text{diag}\{1, -1, -1, -1\}$  in the first order.

The degree of smallness of the input parameter  $g$  might be estimated by using experimental tests of possible violation of the traditional Lorentz invariance. Various relations can be proposed to use in such tests. The important and urgent questions are arising:

*Is today's experimental accuracy sufficient to predict the parameter  $g$ ?*

*How to disentangle the parameter  $g$  from extended relativistic experience?*

The value of  $g$  should characterize the degree to which Lorentz invariance is broken in nature.

**1.6. “Universal” means “geometrical”.** The Finslerian parameter  $g$  is not “bound up” to any particular type of fundamental physical interactions, although the corrections in  $g$  may enter the equations describing any of the fundamental interactions (through, for example,  $g$ -corrections to the metric tensor, tetrads, and connection coefficients). The Finslerian approach does not assume that some fundamental length should be introduced, although there exist numerous analogies among the Finslerian consequences and the consequences of the elementary length-based theories (for example, the occurrence of non-Lorentzian corrections). The parameter  $g$  is not a combination of well-known fundamental physical constants, but, instead, is meant to be a dimensionless fundamental constant. The parameter  $g$  is universal in the sense that it is of pure geometrical origin, that is, the corrections to the pseudo-Riemannian geometry of space-time are introduced through this parameter  $g$ . Merely, *the parameter  $g$  evaluates the degree of Finslerian non-Riemannianity of space-time.*

**1.7. Non-Lorentzian transformations** were considered in several works. The Lorentz transformations and their modifications have been serving over our century to “work-up” the high-energy phenomenology, derive the fundamental physical field equations, and predict new relativistic effects. Despite the general feeling of a high degree of accuracy between predictions and measurements, various modifications, including the well-known cases

- (I) The Robertson Transformations [10]
- (II) The Edward Transformations [18, 22]
- (III) The Mansouri–Sexl Transformations [30]
- (IV) The Tangherlini Transformations [16]

(listed here in the chronological order) have been used [the transformations (I)–(III) have clearly been compared with each other in [39]; a systematic review of various kinematics relations stemmed from the choice (IV) can be found in [28]]. Surprisingly, in a sharp contrast to the approach followed in Einstein’s work [1, 2], in the which STR began with two fundamental invariance principles to derive the required transformations, a lack of profound invariance motivation to favour relativistic treatments is a common feature for the approaches based on the non-Lorentzian transformations (I)–(IV). In fact, the transformations (I)–(IV) have been introduced primarily to reanalyze the role of synchronization procedure [11, 14, 15, 60–61]. No metric function invariant under a member of the set of non-Lorentzian transformations (I)–(IV) has been known.

## 2. The Finslerian Extension of Lorentz Transformations

However, the geometrically-motivated invariance can be retained safely if the Finsler geometry is invoked as a necessary basis thereto. Such a way leads to the following Finslerian special-relativistic kinematic transformations

$$\begin{aligned} R^0 &= \Lambda_0^0 \tilde{R}^0 + \Lambda_1^0 \tilde{R}^1, & R^1 &= \Lambda_0^1 \tilde{R}^0 + \Lambda_1^1 \tilde{R}^1, \\ R^2 &= \Lambda_2^2 \tilde{R}^2, & R^3 &= \Lambda_3^3 \tilde{R}^3, \end{aligned}$$

where the coefficients are

$$\begin{aligned} \Lambda_0^0 &= 1/V(g; v), & \Lambda_0^1 &= \Lambda_1^0 = v/V(g; v), & \Lambda_1^1 &= (1 - g|v|)/V(g; v), \\ \Lambda_2^2 &= \Lambda_3^3 = \sqrt{Q(g; v)}/V(g; v). \end{aligned}$$

The transformations can be inversed to give

$$\begin{aligned} \tilde{R}^0 &= \lambda_0^0 R^0 + \lambda_1^0 R^1, & \tilde{R}^1 &= \lambda_0^1 R^0 + \lambda_1^1 R^1, \\ \tilde{R}^2 &= \lambda_2^2 R^2, & \tilde{R}^3 &= \lambda_3^3 R^3 \end{aligned}$$

with

$$\begin{aligned} \lambda_0^0 &= (1 - g|v|)V(g; v)/Q(g; v), & \lambda_0^1 &= \lambda_1^0 = -vV(g; v)/Q(g; v), \\ \lambda_1^1 &= V(g; v)/Q(g; v), \\ \lambda_2^2 &= \lambda_3^3 = V(g; v)/\sqrt{Q(g; v)}. \end{aligned}$$

The co-version transformations, as related to a four-dimensional momentum  $P_p = \{P_0, P_a\}$ , read

$$\begin{aligned} P_0 &= \Lambda_0^{*0} \tilde{P}_0 + \Lambda_0^{*1} \tilde{P}_1, & P_1 &= \Lambda_1^{*0} \tilde{P}_0 + \Lambda_1^{*1} \tilde{P}_1, \\ P_2 &= \Lambda_2^{*2} \tilde{P}_2, & P_3 &= \Lambda_3^{*3} \tilde{P}_3 \end{aligned}$$

with

$$\begin{aligned} \Lambda_0^{*0} &= 1/W(g; p), & \Lambda_0^{*1} &= \Lambda_1^{*0} = -p/W(g; p), \\ \Lambda_1^{*1} &= (1 + g|p|)/W(g; p), \end{aligned}$$

$$\Lambda_2^{*2} = \Lambda_3^{*3} = \sqrt{Q^*(g; p)}/W(g; p),$$

and for their inverse,

$$\begin{aligned} \tilde{P}_0 &= \lambda_0^{*0} P_0 + \lambda_0^{*1} P_1, & \tilde{P}_1 &= \lambda_1^{*0} P_0 + \lambda_1^{*1} P_1, \\ \tilde{P}_2 &= \lambda_2^{*2} P_2, & \tilde{P}_3 &= \lambda_3^{*3} P_3, \end{aligned}$$

where

$$\begin{aligned} \lambda_0^{*0} &= (1 + g|p|)W(g; p)/Q^*(g; p), & \lambda_0^{*1} &= \lambda_1^{*0} = pW(g; p)/Q^*(g; p), \\ \lambda_1^{*1} &= W(g; p)/Q^*(g; p), \\ \lambda_2^{*2} &= \lambda_3^{*3} = W(g; p)/\sqrt{Q^*(g; p)}. \end{aligned}$$

Here,  $V = F/R^0$ ,  $W = H/P_0$ , and

$$Q = 1 - g|v| - v^2, \quad Q^* = 1 + g|p| - p^2;$$

the notation  $v$  and  $p$  has been used for the three-dimensional velocity and momenta, respectively. Invariance

$$F(g; R) = F(g; \tilde{R}), \quad H(g; P) = H(g; \tilde{P})$$

holds true. *In the limit  $g \rightarrow 0$  the above transformations turn into the conventional Lorentz transformations.*

It should be noted that

$$\Lambda_2^2 \neq 1 \quad \& \quad \Lambda_3^3 \neq 1 \quad \text{whenever} \quad g \neq 0$$

so that we cannot get any generalization if, attempting to generalize the Lorentz transformations in the Finslerian way, we try to retain “as obvious” the principle stating that “the scales perpendicular to motion direction must not be deformed”.

The above formulae entail, in particular, that **the Finslerian addition law for relative velocities** should read

$$v_3 = v_2 \oplus v_1 : \quad v_3 = \frac{v_2 + v_1 - gv_2|v_1|}{1 + v_2v_1}.$$

The inverse subtraction law is

$$v_2 = v_3 \ominus v_1 : \quad v_2 = \frac{v_3 - v_1}{1 - v_1v_3 - g|v_1|}$$

which entails the relations

$$\frac{1}{v_3 \ominus v_1} + \frac{1}{v_1 \ominus v_3} = g \frac{|v_3| - |v_1|}{v_3 - v_1}$$

and

$$\ominus v = -\frac{v}{1 - g|v|}$$

together with the *fundamental group property*

$$(v_1 \oplus v_2) \oplus v_3 = v_1 \oplus (v_2 \oplus v_3).$$

Quite similar Finslerian observations can be made for relativistic momenta.

We observe that the Finslerian extension violates the reciprocity principle (which claims that the velocity of a inertial reference frame  $S$  measured from another inertial reference frame  $S'$  is the opposite of the velocity of  $S'$  measured from  $S$ ), for

$$\ominus v \neq -v \quad \text{whenever} \quad g \neq 0.$$

Owing to the remarkable group property, we are justified in claiming that the Finslerian extension of STR sprung from the FMF  $F(g; R)$  does obey the requirement that the kinematic transformations linking pairs of observers must form a group (see more detail related to the material of the present section in [88]).

### 3. Energy-momentum dependence of respective Finsler type

Considering the Finsler-mass-shell

$$H(g; P_0, |\mathbf{P}|) = m,$$

where  $m$  is the rest mass of the particle, separately in the future-like sector and in the past-like sector, we obtain the two sheets,  $\mathcal{M}^{(+)}(m)$  and  $\mathcal{M}^{(-)}(m)$ , defined respectively by the equations

$$H^{(+)}(g; P_0, |\mathbf{P}|) = m, \quad H^{(-)}(g; P_0, |\mathbf{P}|) = m,$$

which give rise to the energy-momentum functions

$$P_0 = P_0^{(+)}(g; m; |\mathbf{P}|) > 0, \quad P_0 = P_0^{(-)}(g; m; |\mathbf{P}|) < 0.$$

It occurs that, as far as we venture to follow the Finsler-relativistic approach, the sheet  $\mathcal{M}^{(-)}(m)$  ceases to be the mirror image of the sheet  $\mathcal{M}^{(+)}(m)$

Considering the energies

$$E^{(+)} = P_0^{(+)} > 0, \quad E^{(-)} = -P_0^{(-)} > 0,$$

we can find after direct calculations the simple result:

$$\frac{\partial E^{(+)}}{\partial |\mathbf{P}|} = \frac{|\mathbf{P}|}{E^{(+)} + g|\mathbf{P}|}, \quad \frac{\partial E^{(-)}}{\partial |\mathbf{P}|} = \frac{|\mathbf{P}|}{E^{(-)} - g|\mathbf{P}|},$$

These formulae can be used to conclude that, nevertheless,  $E^{(+)}$  and  $E^{(-)}$  are *monotonically increasing functions of  $|\mathbf{P}|$* . The symmetry

$$E^{(+)} \xleftrightarrow{g \leftrightarrow -g} E^{(-)}$$

holds now, instead of the ordinary Lorentzian identity  $E^{(+)} = E^{(-)}$ .

Whenever  $m > 0$ , we can put  $|\mathbf{k}| = |\mathbf{P}|/m$  and examine the low-velocity approximation

$$|\mathbf{k}| \ll m,$$

which yields the following differing energy-momentum dependences for the case of a particle and for the case of an antiparticle:

$$E^{(+)} = 1 + \frac{1}{2}|\mathbf{k}|^2 - \frac{1}{3}g|\mathbf{k}|^3 - \frac{1}{24}(3 + 4g^2)|\mathbf{k}|^4 + \mathcal{O}(g|\mathbf{k}|^5)$$

and

$$E^{(-)} = 1 + \frac{1}{2}|\mathbf{k}|^2 + \frac{1}{3}g|\mathbf{k}|^3 - \frac{1}{24}(3 + 4g^2)|\mathbf{k}|^4 + \mathcal{O}(g|\mathbf{k}|^5)$$

which can be subjected to a due experimental verification, at least in principle, to get estimations on the parameter  $g$ . As a neat particular consequences, we obtain

$$E^{(+)} - E^{(-)} = -\frac{2}{3}g|\mathbf{k}|^3,$$

to the nearest Finslerian order.

From this it follows that we should deal with the two Finsler-relativistic Hamiltonian functions

$$H_1(g; E^{(+)}, |\mathbf{P}|) = \left| E^{(+)} - \frac{|\mathbf{P}|}{g^+} \right|^{G^+/2} \left| E^{(+)} - \frac{|\mathbf{P}|}{g^-} \right|^{-G^-/2}$$



and

$$H_2(g; E^{(-)}, |\mathbf{P}|) = \left| E^{(-)} + \frac{|\mathbf{P}|}{g^+} \right|^{G^+/2} \left| E^{(-)} + \frac{|\mathbf{P}|}{g^-} \right|^{-G^-/2},$$

the respective relevant energy-momentum relations being governed by the equations

$$H_1(g; E^{(+)}, |\mathbf{P}|) = m, \quad H_2(g; E^{(-)}, |\mathbf{P}|) = m.$$

Thus we observe the phenomenon of *the Finslerian splitting* of the ordinary Lorentzian mass-shell, for

*the mass-shells defined in terms of  $H_1$ , and in terms of  $H_2$ , differ from one another unless  $g = 0$ .*

The fact is that the Finsler-relativistic Hamiltonian function  $H(g; P)$  written out in Section 1.4 is no more  $P_0$ – even:

$$H(g; -P_0, \mathbf{P}) \neq H(g; P_0, \mathbf{P}), \quad \text{unless } g = 0.$$

Instead, the function shows the property of *gT-parity*

$$H(-g; -P_0, \mathbf{P}) = H(g; P_0, \mathbf{P}),$$

in addition to the property of *P-parity*

$$H(g; -P_0, -\mathbf{P}) = H(g; P_0, \mathbf{P})$$

that is retained under the given Finslerian extension.

Thus the relativistic Finslerian approach substitutes *the combined gT-symmetry* with the ordinary, Lorentzian and pseudo-Euclidean, *T-parity*.

It can be said also that the Finslerian parameter  $g$  measures the degree of the respective Particle-Antiparticle asymmetry.

#### 4. New Call to Experimenters: Finslerian Neutrino Geometry Versus Lorentzian Light Geometry ?

At present it has become customary to treat the mean rest frame  $\Sigma$  of the universe (the microwave background frame) as the preferred reference frame and use the Earth's speed  $\sim 300 - 400 \text{ km}\cdot\text{s}^{-1}$  with respect to  $\Sigma$  in evaluating estimations for phenomenological parameters of possible violation of the relativistic quantities from their traditional Lorentzian patterns (relevant experimental tests, which are often based on the usage of the modern high-precision laser techniques, are many (see [41–57])). Properly, S.Coleman and S.L.Glashow in the known work [69], have begun with noting a preferred frame to study possible “Neutrino Tests of Special Relativity”.

Moreover, many authors prefer to identify the frame  $\Sigma$  with the Cosmic Substratum, noting that our phenomenological situation in Cosmos just compels attention to a distinct possibility of such media (see [61]). Also, such a media is seemingly an ideal ground for the notion of Cosmic Vacuum Media formed at any local point by averaging over all physical quantum or stochastic fields (similar ideas can be “projected down” from the modern multi-dimensional string theories; see in this respect V.Ammosov and G.Volkov [97] and references therein).

On the other hand, any extension of pseudo-Euclidean square-root metric should obviously affect and deform the Lorentzian mass-shell for high-energy particles. Whence the relativistic and ultra-relativistic particles should be sensitive to the primary geometry

of space-time. Especially, since the mass-shell deformation is accompanied by due deformation of the light cone, we should expect that the neutrinos and antineutrinos, as being entirely uncomposed particles, do feel any possible Finsler-correction *in an utmost and neat way. But the light photons don't at all.*

It can be predicted, therefore, that the neutrinos are quite sensitive to such corrections. Granted the metric is Finslerian rather than pseudo-Euclidean, the neutrinos live in a Finsler geometry rooted in Space-Time *versus* the light photons which, because there are no anti-photons, follows monotonically the pseudo-Riemannian future-oriented cosmic substratum in a sense.

As we pointed out in the preceding section, an accurate-way Finslerian approach entails eventually the asymmetry between the up- and down-sheets of the mass-shell such that, at any fixed rest-mass,

*the mass-shell of a particle is not identical to the mass-shell of its antiparticle unless the Finslerian characteristic parameter,  $g$ , is null.*

Thus the Finsler-relativistic mass-shell at any fixed rest-mass value is now splitting into two different sheets, as being related to particles and antiparticles, so that, on an accurate Finsler-relativistic extension, the Neutrinos and the Antineutrinos occupy some comfortable twin-compartments in a Finsler-relativistic train, while the Photon travels are left intact and outside. *There is no single compartment in the train.*

## 5. Direct Experimental Claim: the neutrino velocity may differ from the light velocity?

*To the first order of magnitude with respect to the Finslerian parameter  $g$ ,*

$$|g| \ll 1,$$

the constants written down in Section 1.4 reduce to

$$G_+ = g_+ = 1 - \frac{g}{2}, \quad G_- = g_- = -1 - \frac{g}{2},$$

$$G^+ = 1 + \frac{g}{2}, \quad G^- = -1 + \frac{g}{2},$$

so that the squared Finsler-relativistic Hamiltonian function takes on the form

$$[H(g; P)]^2 = \left| P_0 - \left(1 - \frac{g}{2}\right)|\mathbf{P}| \right|^{1+\frac{g}{2}} \left| P_0 + \left(1 + \frac{g}{2}\right)|\mathbf{P}| \right|^{1-\frac{g}{2}}$$

that is broken into two cases

$$[H_1(g; P)]^2 = \left( E - \left(1 - \frac{g}{2}\right)|\mathbf{P}| \right)^{1+\frac{g}{2}} \left( E + \left(1 + \frac{g}{2}\right)|\mathbf{P}| \right)^{1-\frac{g}{2}}$$

and

$$[H_2(g; P)]^2 = \left( E + \left(1 - \frac{g}{2}\right)|\mathbf{P}| \right)^{1+\frac{g}{2}} \left( E - \left(1 + \frac{g}{2}\right)|\mathbf{P}| \right)^{1-\frac{g}{2}},$$

extending the ordinary ansatz

$$[H_{pseudo-Euclidean}]^2 = (P_0 - |\mathbf{P}|)(P_0 + |\mathbf{P}|)$$

in an accurate Finslerian  $O(g)$ -way.

If the function  $H_1$  relates conventionally to particles, then  $H_2$  should relate to antiparticles.

Thus, the account for the Finslerian parameter  $g$  does shift the ordinary isotropic cone  $\Sigma : P_0 = \pm|\mathbf{P}|$ , leading actually to the unification  $\Sigma^+ \cup \Sigma^-$  which members,  $\Sigma^+$  and  $\Sigma^-$ , don't mirror one another, for

$$\Sigma^+ : \quad P_0 = (1 - \frac{g}{2})|\mathbf{P}|$$

and

$$\Sigma^- : \quad P_0 = -(1 + \frac{g}{2})|\mathbf{P}|.$$

*How should this splitting display in the elementary particle physics?*

Following the ordinary relativistic treatment of the antiparticle hyperboloid to be the reflection of the past-like hyperboloid into the future region, we ought to recognize that in the Finslerian framework the rest-mass antiparticles should correspond to the cone

$$(\Sigma^+)^* \stackrel{\text{def}}{=} -\Sigma^- : \quad E = (1 + \frac{g}{2})|\mathbf{P}|,$$

which differs from the rest-mass particle-kind cone proper

$$\Sigma^+ : \quad E = (1 - \frac{g}{2})|\mathbf{P}|.$$

This entails immediately the important conclusion that, on calibrating away the speed-of-light to be unity, the velocity values

$$v \stackrel{\text{def}}{=} |\mathbf{P}|/E$$

for neutrinos and antineutrinos should differ from one another according to the rule:

$$v_\nu = 1 + \frac{g}{2}, \quad v_{\bar{\nu}} = 1 - \frac{g}{2}.$$

The middle sum remains intact

$$\frac{v_{\bar{\nu}} + v_\nu}{2} = 1,$$

but the velocity difference

$$v_\nu - v_{\bar{\nu}} = g_\nu.$$

is significant for careful experimental verifications.

In this respect, it will be noted that the recent particle accelerator data seem to support everywhere the estimation  $g_\nu < 10^{-4} - 10^{-5}$  for all charged long-lived elementary particles. The velocity for accelerator-produced neutrinos was studied for the first time in the experiments performed at the Fermilab [95–96], which resulted in the following upper limit at 95% CL:

$$|v_\nu/c - v_{\bar{\nu}}/c| < 0.7 \cdot 10^{-4}.$$

As was pointed out recently in the work by V.Ammosov and G.Volkov [97], the expected pending neutrino experiments can get sensitive enough to measure the neutrino-antineutrino velocity differences  $\sim 10^{-6}$  for short baseline experiments and  $\sim 10^{-9}$  for long baseline experiments, and that these sensitivities can directly transform into the  $g_\nu$

sensitivity. Since, as we have argued in Section 4, the photons  $\gamma$  don't feel any Finsler geometry, we have for them formally  $g_\gamma = 0$ .

Certainly, the STR cannot consent to draw any distinction between the neutrino velocity and the light velocity (in vacuum). Whence any positive outcome of such-type experiments would put reliable limits on all the body of the STR.

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